A ribbon knot which is not a symmetric union

K-OS Seminar

Joint work with Teruaki Kitano and Yuta Nozaki

1er juillet 2026

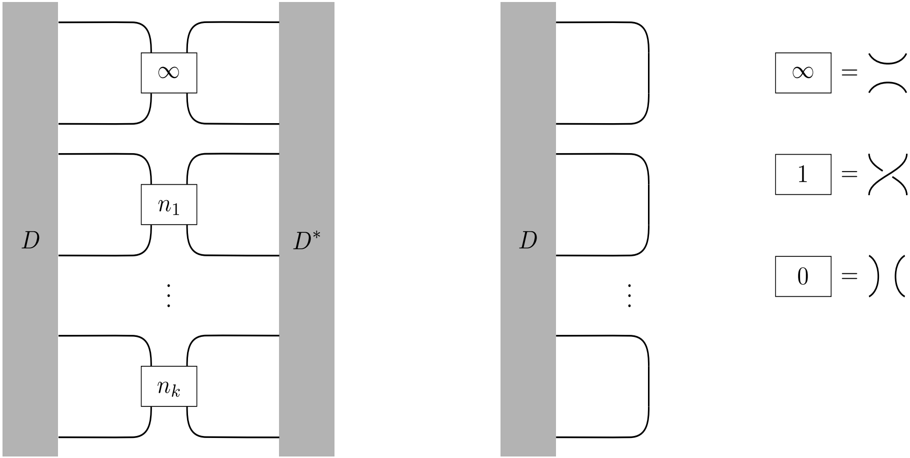


FIGURE : Symmetric union $(DUD^*)(n_1, \ldots, n_k)$

The knot diagram $(DUD^*)(n_1, \ldots, n_k)$ obtained from $D\sharp D^*$ is called the **symmetric union** of the diagram D and D^*.

The knot type of the diagram D is called the **partial knot** K_D of the symmetric union.

If K admits a symmetric union diagram $(DUD^*)(n_1, \ldots, n_k)$ we say that it is a **symmetric union presentation** of K with **partial knot** K_D.

The trivial tangles where the twists take place are called the twist regions.

The symmetric union construction depends on both the diagram D and the location of the twist regions.

This construction has been introduced by Kinoshita and Terasaka (1957) and generalized by C. Lamm (2000).

A symmetric unions is a ribbon knot.

$D\sharp D^*$ bounds a ribbon disc in S^3 swept out by arcs between symmetric points.

Twisting along the symmetry axis induces half-twists of this ribbon disk.

Getting a symmetric union presentation for a knot K is a way to show that it is a ribbon knot.

Symmetric unions are omnipresent among prime ribbon knots with ≤ 12 crossings : up to 10 crossings they are all symmetric unions, and for 11 and 12 crossings this is true for all but 15 ribbon knots [Lamm 2019].

All ribbon 2-bridge knots are symmetric unions [C. Lamm 2021].

Problem (C. Lamm 2000)

Does every ribbon knot admit a symmetric union presentation ?

No obstruction is known for a ribbon knot to be a symmetric union.

Moreover the number of twist regions required to realise a given ribbon knot as a symmetric union can be arbitrarily large [V. Brejevs-P. Feller 2024].

We give a negative answer to Lamm's question by exhibiting a ribbon Montesinos knot which does not admit a symmetric union presentation.

For a knot $K \subset S^3$ we denote by $\Sigma_2(K)$ its 2-fold branched covering.

$H_1(\Sigma_2(K);\mathbb{Z}) \cong \mathbb{Z}/a_1\mathbb{Z} \oplus \cdots \oplus \mathbb{Z}/a_r\mathbb{Z}$ where $r = \operatorname{rank} H_1(\Sigma_2(K);\mathbb{Z})$

and $1 < a_i$ divides a_{i+1} for $1 \leq i \leq r-1$.

Proposition

Let K be a knot which admits a symmetric union presentation with partial knot K_D, then :

(i) $|H_1(\Sigma_2(K);\mathbb{Z})| = |H_1(\Sigma_2(K_D);\mathbb{Z})|^2$.

(ii) $\operatorname{rank} H_1(\Sigma_2(K_D);\mathbb{Z}) \leq \operatorname{rank} H_1(\Sigma_2(K);\mathbb{Z}) \leq 2 \operatorname{rank} H_1(\Sigma_2(K_D);\mathbb{Z})$.

(iii) *There is a π_1-surjective map $f : \Sigma_2(K) \to \Sigma_2(K_D)$ of degree zero.*

The proof of (i) and (ii) follows from C. Lamm (2000), by using the Goeritz matrix G_K for a symmetric union diagram of K to present $H_1(\Sigma_2(K);\mathbb{Z})$.

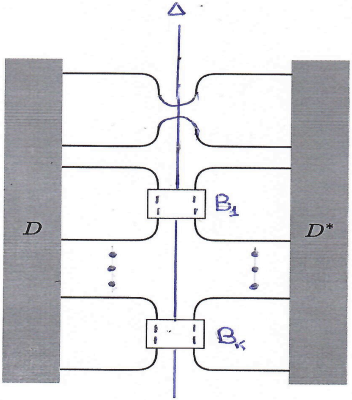
C. Lamm shows that G_K is equivalent to $\begin{pmatrix} G_D & -A \\ 0 & -G_D \end{pmatrix}$, where G_D is the Goeritz matrice for the diagram D of the partial knot K_D.

$|H_1(\Sigma_2(K);\mathbb{Z})| = \det G_K = (\det G_D)^2 = |H_1(\Sigma_2(K_D);\mathbb{Z})|^2$ yields (i)

For any field $\mathbb{F}$, one gets $\dim_{\mathbb{F}} H_1(\Sigma_2(K);\mathbb{F}) \leq 2\dim_{\mathbb{F}} H_1(\Sigma_2(K_D);\mathbb{F})$.

If $H_1(\Sigma_2(K);\mathbb{Z}) \cong \mathbb{Z}/a_1\mathbb{Z} \oplus \cdots \oplus \mathbb{Z}/a_r\mathbb{Z}$ where $r = \text{rank}(H_1(\Sigma_2(K);\mathbb{Z}))$ and $1 < a_i$ divides a_{i+1} for $1 \leq i \leq r-1$.

Letting $\mathbb{F} = \mathbb{F}_p$ for a prime p dividing a_1 gives the inequality (ii)


Δ
B_1
B_k
D
D^*

Proof of (iii)

There is a reflection ρ through a sphere S containing the axis Δ which preserves the connected sum $K_D \sharp K_{D^*} = (DUD^*)(0, \ldots, 0)$ and the trivial tangles $B_1, \ldots, B_k$.

ρ lifts to an orientation reversing involution $\tilde{\rho}$ on $\Sigma_2(K_D) \sharp \Sigma_2(K_D^*)$.

$\tilde{\rho}$ exchanges the punctured sides $\Sigma_2^0(K_D)$ and $\Sigma_2^0(K_D^*)$.

Let the 2-fold covering projections $q\colon \Sigma_2(K_D \sharp K_{D^*}) \to (S^3, K_D \sharp K_{D^*})$, $q_D\colon \Sigma_2(K_D) \to (S^3, K_D)$, and $q_D^*\colon \Sigma_2(K_D^*) \to (S^3, K_D^*)$.

For $i = 1, \cdots, k$, $V_i = q^{-1}(B_i)$ is a $\tilde{\rho}$-invariant solid torus.

Let $X = \Sigma_2(K_D \sharp K_{D^*}) \setminus \bigsqcup_{i=1}^{k} \operatorname{int} V_i$.

X is invariant by the involution $\tilde{\rho}$ which exchanges the sides $\Sigma_2^0(K_D) \cap X$ and $\Sigma_2^0(K_D^*) \cap X$.

X is a submanifold of $\Sigma_2(K)$ and $\Sigma_2(K)$ is obtained by Dehn filling of ∂X.

Define a map $f\colon X \to \Sigma_2^0(K_D) \cap X$ by :

$f(x) = x$ if $x \in \Sigma_2^0(K_D) \cap X$ and $f(x) = \tilde{\rho}(x)$ otherwise.

By construction, degree of $f = 0$.

By construction $f_*\colon \pi_1 X \twoheadrightarrow \pi_1(\Sigma_2^0(K_D) \cap X)$ is surjective.

For $i = 1, \cdots, k$, $\tilde{B}_i = q_D^{-1}(B_i)$ is a 3-ball.

$$\Sigma_2^0(K_D) = \left(\Sigma_2^0(K_D) \cap X\right) \cup \bigsqcup_{i=1}^{k} \tilde{B}_i.$$

Hence $\pi_1(\Sigma_2^0(K_D) \cap X) = \pi_1 \Sigma_2^0(K_D) = \pi_1 \Sigma_2(K_D)$

f sends the boundary tori $T_i \subset \partial X$ to the 2-spheres $\partial \tilde{B}_i$, for $1 \leq i \leq k$.

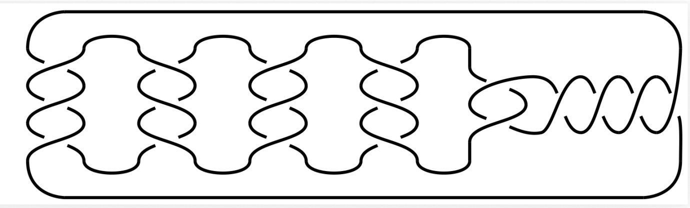
Therefore, f can be extended to a degree zero and π_1-surjective map

$$f \colon \Sigma_2(K) = X \cup \bigsqcup_{i=1}^{k} V_i \to \left(\Sigma_2^0(K_D) \cap X\right) \cup \bigsqcup_{i=1}^{k} \tilde{B}_i = \Sigma_2^0(K_D).$$

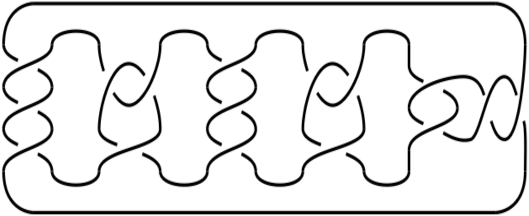
by sending the solid tori V_i to the 3-balls $\tilde{B}_i$, for $1 \leq i \leq k$.

Theorem (B-Kitano-Nozaki)

The Montesinos knot $K = K(\frac{1}{3}, -\frac{1}{3}, \frac{1}{3}, -\frac{1}{3}, \frac{9}{2})$ is a ribbon knot which does not admit a symmetric union presentation.



$K(\frac{1}{3}, -\frac{1}{3}, \frac{1}{3}, -\frac{1}{3}, \frac{9}{2}) = K(\frac{1}{3}, \frac{2}{3}, \frac{1}{3}, \frac{2}{3}, \frac{5}{2})$ is alternating with 16 crossings.



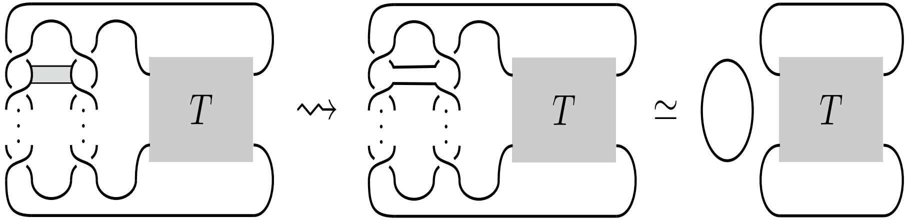
The theorem is true also for the other Montesinos knots obtained by permuting the rational tangles, namely (up to mirror images) :

$K(\frac{1}{3}, \frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}, \frac{9}{2})$; $K(\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}, \frac{1}{3}, \frac{9}{2})$; $K(-\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, -\frac{1}{3}, \frac{9}{2})$.

Lemma

The knot $K = K(\frac{1}{3}, -\frac{1}{3}, \frac{1}{3}, -\frac{1}{3}, \frac{9}{2})$ *is ribbon.*

Attaching a band between the adjacent strands with crossing number 3 and -3 as shown in Figure yields the disjoint union of two unknots and of the ribbon Montesinos knot $K(\frac{9}{2}) = \mathfrak{b}(9,2) = 6_1$.



Proof of the Theorem.

The proof that $K = K(\frac{1}{3}, -\frac{1}{3}, \frac{1}{3}, -\frac{1}{3}, \frac{9}{2})$ does not admit a symmetric union presentation follows by contradiction from two propositions :

Proposition (A)

If $K = K(\frac{1}{3}, -\frac{1}{3}, \frac{1}{3}, -\frac{1}{3}, \frac{9}{2})$ admits a symmetric union presentation with partial knot K_D, then $\Sigma_2(K_D)$ must be aspherical.

Proposition (B)

If $K = K(\frac{1}{3}, -\frac{1}{3}, \frac{1}{3}, -\frac{1}{3}, \frac{9}{2})$ admits a symmetric union presentation with partial knot K_D, then $\Sigma_2(K_D)$ cannot be aspherical.

We compute the first homology of $\Sigma_2(K)$ and $\Sigma_2(K_D)$.

$\Sigma_2(K)$ is the Seifert fibred manifold $V(0; \frac{9}{2}; \frac{1}{3}, -\frac{1}{3}, \frac{1}{3}, -\frac{1}{3}, \frac{9}{2})$

with rational Euler number $e_0 = \frac{9}{2}$ and base orbifold $S^2(3,3,3,3,2)$.

Then $|H_1(\Sigma_2(K); \mathbb{Z})| = |e_0| 3^4 \times 2 = 9 \times 81$.

Lemma

$H_1(\Sigma_2(K); \mathbb{Z}) \cong \mathbb{Z}/3\mathbb{Z} \oplus \mathbb{Z}/3\mathbb{Z} \oplus \mathbb{Z}/81\mathbb{Z}$. Hence rank *$H_1(\Sigma_2(K); \mathbb{Z}) = 3$.*

In $H_1(\Sigma_2(K); \mathbb{Z})$ let x, y, u, v, represent the singular fibers of order 3,

z the singular fiber of order 2 and h the regular fiber, Then $H_1(\Sigma_2(K);\mathbb{Z})$ is generated by x, y, u, v, z, h subject to the relations :

$$3x + h = 3y - h = 3u + h = 3v - h = 2z + 9h = x + y + z + u + v = 0.$$

$$H_1(\Sigma_2(K);\mathbb{Z})/\langle x\rangle = \langle y, u, v, z \,|\, 3y = 3u = 3v = 2z = y + u + v + z = 0$$

$z = 0$ and $H_1(\Sigma_2(K);\mathbb{Z})/\langle x\rangle = \langle y, v \,|\, 3y = 3v = 0\rangle \cong \mathbb{Z}/3\mathbb{Z} \oplus \mathbb{Z}/3\mathbb{Z}$

$\langle x\rangle \cong \mathbb{Z}/81\mathbb{Z}$

$$1 \to \mathbb{Z}/81\mathbb{Z} \cong \langle x\rangle \to H_1(\Sigma_2(K);\mathbb{Z}) \twoheadrightarrow \langle\overline{x+y}\rangle \oplus \langle\overline{x+v}\rangle = \mathbb{Z}/3\mathbb{Z} \oplus \mathbb{Z}/3\mathbb{Z}.$$

The exact sequence splits and $H_1(\Sigma_2(K);\mathbb{Z}) \cong \mathbb{Z}/3\mathbb{Z} \oplus \mathbb{Z}/3\mathbb{Z} \oplus \mathbb{Z}/81\mathbb{Z}$

Corollary

$H_1(\Sigma_2(K_D);\mathbb{Z}) = \mathbb{Z}/3\mathbb{Z} + \mathbb{Z}/9\mathbb{Z}$ *or* $\mathbb{Z}/3\mathbb{Z} + \mathbb{Z}/3\mathbb{Z} + \mathbb{Z}/3\mathbb{Z}$.

$2\,\mathrm{rank}\, H_1(\Sigma_2(K_D);\mathbb{Z}) \geq \mathrm{rank}\, H_1(\Sigma_2(K);\mathbb{Z}) = 3.$

So $2 \leq \mathrm{rank}\, H_1(\Sigma_2(K_D);\mathbb{Z}) \leq \mathrm{rank}\, H_1(\Sigma_2(K);\mathbb{Z}) = 3.$

$|H_1(\Sigma_2(K_D);\mathbb{Z})| = \sqrt{|H_1(\Sigma_2(K_D);\mathbb{Z})|} = 27.$

Hence $H_1(\Sigma_2(K_D);\mathbb{Z}) \cong \mathbb{Z}/3\mathbb{Z} + \mathbb{Z}/9\mathbb{Z}$ or $\mathbb{Z}/3\mathbb{Z} + \mathbb{Z}/3\mathbb{Z} + \mathbb{Z}/3\mathbb{Z}$.

Proof of Proposition A

Let assume that $\Sigma_2(K_D)$ is not aspherical.

Either $\pi_1\Sigma_2(K_D)$ is finite or $\Sigma_2(K_D)$ is a non-trivial connected sum.

If $\pi_1\Sigma_2(K_D)$ is finite then $H_1(\Sigma_2(K_D);\mathbb{Z})$ is cyclic, because it is a Lens space or a Seifert fibred manifold over $S^2(2,3,3)$ or $S^2(2,3,5)$.

rank $H_1(\Sigma_2(K_D);\mathbb{Z}) \geq 2 \Rightarrow \Sigma_2(K_D)$ must be a non trivial connected sum.

$\pi_1\Sigma_2(K_D)$ is a non-trivial free product (proof of Poincaré conjecture).

Hence $\pi_1\Sigma_2(K_D)$ is centerless $\Rightarrow$ the epimorophism

$f_*\colon \pi_1\Sigma_2(K) \twoheadrightarrow \pi_1\Sigma_2(K_D)$ factorizes through $\pi_1^{orb}S^2(2,3,3,3,3)$.

$\pi_1^{orb} S^2(2,3,3,3,3)$ surjects onto each factor of the free product $\pi_1 \Sigma_2(K_D)$.

$\pi_1^{orb} S^2(2,3,3,3,3) = \langle z, x, y, u, v \mid z^2 = x^3 = y^3 = v^3 = u^3 = zxyuv = 1 \rangle$ is generated by torsion elements, so no factor can be torsion free.

Hence each factor is the fundamental group of a prime rational homology sphere with torsion elements.

It must be finite and the manifold is a Lens space or Seifert fibred over $S^2(2,3,3)$ or $S^2(2,3,5)$.

So $\Sigma_2(K_D)$ is a connected sum of n_1 Lens space and n_2 Seifert fibred manifolds over $S^2(2,3,3)$ or $S^2(2,3,5)$.

$n_1 + 2n_2 \leq \operatorname{rank} \pi_1^{orb} S^2(2,3,3,3,3) \leq 4$.

$\mathbb{Z}/3\mathbb{Z} \oplus \mathbb{Z}/3\mathbb{Z} \oplus \mathbb{Z}/3\mathbb{Z} = (\pi_1^{orb} S^2(2,3,3,3,3))^{ab}$ surjects onto

$H_1(\Sigma_2(K_D);\mathbb{Z}) \cong \mathbb{Z}/3\mathbb{Z} + \mathbb{Z}/9\mathbb{Z}$ or $\mathbb{Z}/3\mathbb{Z} + \mathbb{Z}/3\mathbb{Z} + \mathbb{Z}/3\mathbb{Z}$

Hence $H_1(\Sigma_2(K_D);\mathbb{Z}) \cong \mathbb{Z}/3\mathbb{Z} \oplus \mathbb{Z}/3\mathbb{Z} \oplus \mathbb{Z}/3\mathbb{Z}$.

Then the only possibilities for $\Sigma_2(K_D)$ are :

(i) $\Sigma_2(K_D) = L(3,1)\sharp L(3,1)\sharp L(3,1)$ or

(ii) $\Sigma_2(K_D) = L(3,1)\sharp L(3,1)\sharp M$, with $\pi_1 M$ finite and $H_1(M;\mathbb{Z}) \cong \mathbb{Z}/3\mathbb{Z}$.

So $\pi_1^{orb} S^2(2,3,3,3,3) \twoheadrightarrow \pi_1\Sigma_2(K_D) \twoheadrightarrow \mathbb{Z}/3\mathbb{Z} \star \mathbb{Z}/3\mathbb{Z} \star \mathbb{Z}/3\mathbb{Z} \to 1$

This epimorphism factorizes through $\pi_1^{orb} S^2(3,3,3,3)$ because the element of order 2 in $\pi_1^{orb} S^2(2,3,3,3,3)$ is killed.

Hence $\pi_1^{orb} S^2(3,3,3,3) \twoheadrightarrow \pi_1 \Sigma_2(K_D) \twoheadrightarrow \mathbb{Z}/3\mathbb{Z} \star \mathbb{Z}/3\mathbb{Z} \star \mathbb{Z}/3\mathbb{Z} \to 1$

SInce $\pi_1^{orb} S^2(3,3,3,3)$ is generated by 3 elements of order 3, there is an epimorphism from $\mathbb{Z}/3\mathbb{Z} \star \mathbb{Z}/3\mathbb{Z} \star \mathbb{Z}/3\mathbb{Z}$ onto $\pi_1^{orb} S^2(3,3,3,3)$.

By composing the two epimorphisms one gets an epimorphism :

$\mathbb{Z}/3\mathbb{Z} \star \mathbb{Z}/3\mathbb{Z} \star \mathbb{Z}/3\mathbb{Z} \twoheadrightarrow \pi_1^{orb} S^2(3,3,3,3) \twoheadrightarrow \pi_1 \Sigma_2(K_D) \twoheadrightarrow$
$\mathbb{Z}/3\mathbb{Z} \star \mathbb{Z}/3\mathbb{Z} \star \mathbb{Z}/3\mathbb{Z} \to 1$

This is an isomorphism since $\mathbb{Z}/3\mathbb{Z} \star \mathbb{Z}/3\mathbb{Z} \star \mathbb{Z}/3\mathbb{Z}$ is hopfian.

This implies that $\mathbb{Z}/3\mathbb{Z} \star \mathbb{Z}/3\mathbb{Z} \star \mathbb{Z}/3\mathbb{Z}$ is isomorphic to $\pi_1^{orb} S^2(3,3,3,3)$ which is impossible.

Proof of the Proposition B

Proposition B follows from the existence of a π_1-surjective map $f : \Sigma_2(K) \to \Sigma_2(K_D)$ of degree zero and the next proposition :

Proposition (C)

A Seifert fibred manifold M with base $B_M = S^2(2,3,3,3,3)$ does not admit any π_1-surjective map of degree zero $f : M \to N$, onto a closed, aspherical, orientable 3-manifold N.

The proof uses arguments analogous to those used by A. Reid, S. Wang and Q. Zhou (2002) to study π_1-surjective maps of degree zero between aspherical closed Seifert 3-manifolds.

Let assume that such a π_1-surjective map of degree zero exists.

Lemma (1)

1) N carries a Seifert fibration over the base B_N with underlying space S^2. and at least three singular points.

2) The epimorphism $f_* \colon \pi_1 M \to \pi_1 N$ verifies $f_*(h_M) = h_N^d$ with $d \neq 0$, where $h_M \in \pi_1 M$ and $h_N \in \pi_1 N$ correspond to regular fibers of the Seifert fibrations of M and N.

Proof

$\pi_1 M$ has an infinite cyclic center $Z_M = \langle h_M \rangle$.

N aspherical $\Rightarrow \pi_1 N$ torsion free $\Rightarrow f_*(h_M) \neq 1$ because $\pi_1(M)/Z_M$ is generated by torsion elelements.

$f_*(h_M) \neq 1$ belongs to the center of $\pi_1 N$ which is not trivial.

By Casson-Jungreis and Gabai N is Seifert fibred over a base B_N.

$f_* \colon \pi_1 M \twoheadrightarrow \pi_1 N$ induces an epimorphism $|\bar{f}_*| \colon \pi_1 |B_M| \to \pi_1 |B_N|$.

Therefore $\pi_1 |B_N| = \{1\}$ and $|B_N| = S^2$.

$f_*(h_M) \neq 1$ belongs to the infinite cyclic center $Z_N = \langle h_N \rangle \cong \mathbb{Z}$ of $\pi_1 N$.

Hence $f_*(h_M) = h_N^d$ with $d \neq 0$.

N has at least 3 singular fibers since N is aspherical.

Lemma (Reid-Wang-Zhou)

The π_1-surjective map of degree zero $f: M \to N$ can be homotoped to a fiber-preserving map which misses a fibred tubular neighborhood V_0 of a regular fiber of N.

Proof

N is aspherical with a unique, up to homotopy, Seifert fibration with orientable basis, and $f_*(h_M) = h_N^d$ with $d \neq 0$. Then the map f can be homotoped to a fiber preserving map, still called f.

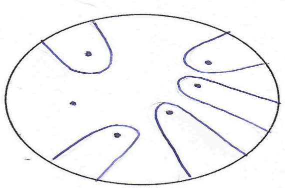
Now the lemma is a fibred version of Kneser's theorem for surfaces, using that the map induced by f between the bases of M and N has degree zero, and thus is homotopic to a map which misses a disk.

Proof of Proposition C

Let $N' = N \setminus \mathrm{int}(V_0) \subset N$ and $B_{N'} = B_N \setminus \mathrm{int}(D_0)$ be the base of N'.

Let $\mathcal{A}$ be a finite collection of disjoint, essential, non-parallel, vertical and properly embedded annuli in N' which cuts N' into Seifert fibered solid tori

Such a collection $\mathcal{A}$ is not empty because the orbifold $B_{N'}$ has at least three singular points, like B_N.



One can homotop f so that $\mathcal{T} = f^{-1}(\mathcal{A})$ is incompressible in M.

$\mathcal{T} \neq \emptyset$ otherwise $f_*(\pi_1 M)$ would be cyclic.

M rational homology sphere with an orientable base $\Rightarrow \mathcal{T}$ cannot have a component transverse to the Seifert fibration.

One can homotop f so that $\mathcal{T}$ is a non-empty finite collection of incompressible vertical tori in M.

Given a torus $T \in \mathcal{T}$, it splits M into two Seifert fibered submanifolds S_1 and S_2 with incompressible boundary $\partial S_1 = \partial S_2 = T$.

Since $f(T) \subset A \in \mathcal{A}$ and $f_*(h_M) = h_N^d$ with $d \neq 0$, $\ker(f_{|T})_* \cong \mathbb{Z}$ is generated by a section s of the Seifert fibration on T.

Therefore, for each Seifert fibered submanifold S_i ($i = 1, 2$) with boundary T, the induced homomorphism $f_*\colon \pi_1 S_i \to \pi_1 N$ factors through $\pi_1 S_i(s)$, where $S_i(s)$ is obtained by Dehn filling ∂S_i along the section s.

For $i = 1, 2$, $S_i(s)$ is a closed Seifert fibered manifold such that the core of the filling is a regular fiber of $S_i(s)$, and hence $S_i(s)$ has the same number of singular fibers as S_i.

Moreover, $f_*(\pi_1 S_i)$ must be infinite because $f_*(h_M)$ is of infinite order in $\pi_1 N$ and belongs to $f_*(\pi_1 S_i)$.

$f_*\colon \pi_1 S_i \to \pi_1 N$ factors through $\pi_1 S_i(s) \Rightarrow \pi_1 S_i(s)$ is infinite.

Assume that the orbifold base B_M is $S^2(2, 3, 3, 3, 3)$.

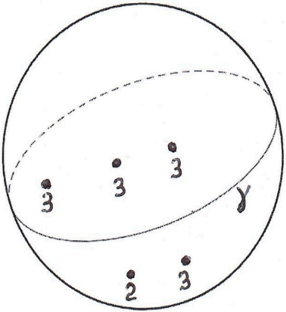
The vertical torus T projects to a simple closed curve γ on B_M, which splits B_M into two disks, each containing at least two singular points, because T is incompressible in M.

There are only two possible distributions of the singular points between the two disks : $\{2, 3\} \cup \{3, 3, 3\}$ or $\{3, 3\} \cup \{2, 3, 3\}$.

Therefore one of the Seifert manifolds $S_1(s)$ or $S_2(s)$ has either $S^2(2, 3)$ or $S^2(2, 3, 3)$ as a base.

In both cases its fundamental group is finite.

This shows that the orbifold base B_M cannot be $S^2(2, 3, 3, 3, 3)$.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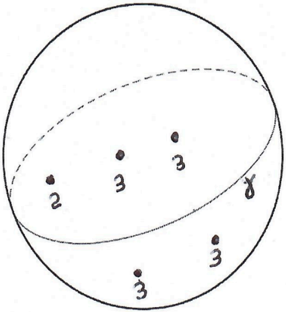
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